THEOREM 5. (CHARACTERIZATION OF (s,ϕ)-FPS)[8]
Consider $s \in [0,\infty]$, a $[0,1]$-automorphism ϕ and an (s,ϕ)-FPS (P,I,J) on a set of alternatives A. Then the following equality holds

$$(P,I,J) = (R \cap_\phi^s \mathrm{co}_\phi R^t,\ R \cap_\phi^s R^t,\ \mathrm{co}_\phi R \cap_\phi^s \mathrm{co}_\phi R^t)$$

with $R = P \cup_\phi^\infty I$.

6 Conclusion

We have described our itinerary in attempting to define, to construct and – above all – to characterise fuzzy preference structures. Various pitfalls had to be avoided, and only through the introduction of non-trivial conditions, fuzzy preference structures could finally be characterised. These fundamental results will undoubtedly foster the further development of both theory and applications of fuzzy preference modelling.

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THE PROBLEM OF DEFINING NON DETERMINISTIC AGGREGATION OPERATORS

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Some formalizations of physical or human decision processes must take into account some sort of non-determinism ("free will"). Such (generally complex) processes always contain some kind of aggregation procedures as building blocks. Thus, non determinism is a problem that must be recognized and dealt with for aggregation operators. We will then introduce here the notion of non-deterministic aggregation operator.

Keywords: Fuzzy Connectives, Fuzzy Sets, Aggregation Operators.

1 Introduction and Preliminaries

"The question of free will [...] is a real issue here, rather than an imagined one, but it is profound and hard to formulate adequately. The issue of *determinism* in physical theory is important, but [...] it is only part of the story." (see [10], pp.170). This quote from Penrose's famous book helps us introduce the motivation behind this paper. Many real-life problems are solved by means of some information aggregation procedures to be designed and used in an "intelligent" manner. In particular, we will follow here the formalizations of intelligent aggregation of information given and characterized by the authors in [3,4,5,6,7] (see also [11,12,13,14] for related work)

Information is passed to an aggregation operator as an ordered sequence of real numbers, which without loss of generality can be supposed to belong to the unit interval.

Aggregation operators are quite simple maps. Formally, an aggregation operator of dimension n, is any mapping

$$\phi : [0,1]^n \to [0,1].$$

We can also have hierarchical aggregations, that is to say aggregations of chunks of information which in turn represent aggregated information. The practical consequences of such hierarchical aggregations are quite interesting.

If we have aggregation maps of very big dimensions, hierarchical aggregations will allow us to deal with sub-aggregation operators of smaller dimensions, whose computational jobs can be parallelised. Thus, it will be possible to obtain a significant speed up of the whole aggregation process.

For instance, hierarchical aggregation procedures for individual preferences are defined by means of a basic classification of the individuals. The set of individual is divided into groups, in such a way that each individual is present in at least one of these groups. Then partial amalgamations of opinions within each group are to be amalgamated into one global amalgamation.

2 Non deterministic aggregation operators

Similarly to Turing Machine, a non deterministic aggregation operator is such that the same input can produce different outputs. For Turing Machine non determinism has the intended meaning of computational power: in one step the machine can non deterministically choose any of the possible outcomes. Non deterministic algorithms "guess" the best outcome among all possible ones.

Analogously, a non deterministic aggregation operator can be defined in the following way:

- we have a set M of mappings

$$\phi : [0,1]^n \to [0,1]$$

- and a non deterministic choice function (Turing Machine) η such that

$$\Phi(x_1,\ldots,x_n) = \eta(\{\phi(x_1,\ldots,x_n)|\phi \in M\})$$

Such aggregation operators will be denoted by $\Phi(M)$. Thus, by definition

$$\Phi(M)(x_1,\ldots,x_n) = \eta(\{\phi(x_1,\ldots,x_n)|\phi \in M\}).$$

For aggregation operators non determinism will have the intended meaning of "intelligence" power. We can think of it as a process that in one step tests all the possible operators of the family M and chooses the best one.

It is now quite natural to introduce the concepts of non-deterministic T-norms and T-conorms and OWA operators.

DEFINITION 1 *A non deterministic T-norm is a non deterministic aggregation operator $\Phi(M)$ such that M is a set of T-norms.*

Analogously, a non deterministic T-conorm is a non deterministic aggregation operator $\Phi(M)$ such that M is a set of T-conorms.

Finally, a non deterministic OWA operator is a non deterministic aggregation operator $\Phi(M)$ such that M is a set of OWA operators.

The above definitions clearly extend the traditional definitions which are obtained for $|M| = 1$. Moreover, we can characterize non deterministic aggregation operators as finite, discrete and continuous according to the cardinality of M.

In an extended version of this paper, we will comment upon such different families.

3 More on OWA operators.

A significative measure associated with OWA operators is the orness which estimates how close an OWA operator is to the *max* operator. It is defined as

$$orness(\phi) = \frac{1}{n-1}\sum_{i=1}^{n}(n-i)w_i.$$

Dual to the measure of *orness* is the measure of *andness* defined as $andness(\phi) = 1 - orness(\phi)$, which therefore measures how close an OWA operator is to the min operator.

How do we extend such definitions to non deterministic OWA operators ? If we see the degrees of orness and andness respectively as degrees of optimism and pessimism, then we immediately have (for M finite)

$$\begin{aligned} orness(\Phi(M)) &= \max\{orness(\phi)|\phi \in M\} \\ andness(\Phi(M)) &= \min\{andess(\phi)|\phi \in M\} \end{aligned}$$

Therefore, likewise deterministic OWA operators, we have

$$orness(\Phi(M)) = 1 - andness(\Phi(M)).$$

If M is infinite, we substitute min and max with inf and sup and the result does not change.

4 Example

Very often in life we are faced with the problem of making a decision based on some imprecise knowledge. Typical cases of this kind occur in the medical field. In an emergency room of a hospital doctors have to decide whether or not to admit a patient in the hospital (see [8]). Their knowledge is what they have learned in Medical School (enriched by the experience gained at work) and the symptoms they observe in the patient. For instance, they may see that the patient "has a low fever", "is sweating a lot" and "feels a small pain in the chest". By introducing ad hoc membership functions, one can use standard fuzzy inference mechanisms to come up with a value of admissibility for the patient. Then based on this value (typically introducing a security threshold parameter) one can decide whether to admit the patient or not. However, since in this case we are dealing with human lives, we need to be reasonably sure that the used membership functions are well tuned and in turn that the inference mechanisms and the system of fuzzy rules with a non-fuzzy conclusion we are working with has been validated (see [2]). How can we do that ?

A theoretical framework for tuning membership functions inside a non-fuzzy classification system was proposed in [1].

It is assumed that a system of linguistic classification rules is given by an expert. However, the expert does not provide any definition either of the fuzzy sets or of the aggregation operators is using. Common sense tells us that a medical student may be told that a patient with "high fever" and "strong abdominal pain" must be admitted to the hospital, though certainly he/she is not told that a body temperature of 38 centigrades has a degree of membership .4 to the fuzzy set "high fever". In [1] it is built an induction system which learns the membership functions of the fuzzy predicates occurring in the system by observing positive and negative examples.

Let us briefly see how is the system formally defined.

Let C be a given concept class over a universe $\mathcal{U}$. For the time being we will suppose that $\mathcal{U} \subseteq \Re^k$ for some integer k. Therefore, each element of $\mathcal{U}$ is a k-tuple of real numbers whose intended meaning is to characterize specific subparts of the system under study.

Let $C \in \mathcal{C}$, i.e. C is a concept belonging to the coucept class $\mathcal{C}$, and suppose a system S_m of classification rules for C is given in the following form:

$$\begin{array}{lll} Q_1^1(p_{Q_1^1}(x)) & \wedge \cdots \wedge Q_{n_1}^1(p_{Q_{n_1}^1}(x)) \rightarrow C(x) & \\ \cdots & \cdots \qquad\qquad \cdots & (4.1) \\ Q_1^m(p_{Q_1^m}(x)) & \wedge \cdots \wedge Q_{n_m}^m(p_{Q_{n_m}^m}(x)) \rightarrow C(x) & \end{array}$$

where

- the predicates Q_i^j are taken from a set $\mathcal{P}=\{Q_1,\ldots,Q_n\}$ of given unary predicates;
- for all Q_j, p_{Q_j} is a projection function that returns the parameter which is of significance for Q_j.
- for every component k, the predicates $Q_1,\ldots,Q_{m_k}$ corresponding to k (i.e. such that p_{Q_i} returns the value of the k-th component) define a linguistic order.

Notice that

- it is possible that the same predicate occurs in different rules;
- the unknown membership functions are *convex* (see [9], pp. 17). Thus, they do not have local maxima. Moreover, for what concerns the linguistic order, the convexity hypothesis implies that for every predicate Q the set $\{x|Q(x) > \alpha\}$ is an interval of the real line for every α. So, we can suppose that if $j > i$ then for every $0 \leq \alpha \leq 1$ we have $\{x|Q_i(x) > \alpha\} \prec \{x|Q_j(x) > \alpha\}$, where $\prec$ is the standard order relation on real intervals:

$$[a,b] \prec [c,d] \text{ iff } a \leq c \text{ and } b \leq d.$$

The intuitive meaning of the rule system is that a given individual x is classified as a positive example for the concept C if one or more of the rule antecedents are true for x.

Therefore we can say that $x \in \mathcal{U}$ is a member of the concept C i.e. $C(x)$ is true if and only if

$$\tau\left(\bigvee_{j=1}^{m}\bigwedge_{i=1}^{n_j} Q_i^j(p_{Q_i^j}(x))\right) = 1$$

where τ is the the truth function. Since we are allowing some predicates in $\mathcal{P}$ to be fuzzy whereas C is not we make the following assumptions:

- A security parameter $0 < \theta < 1$ is given;
- $C(x)$ is θ-true if and only if

$$\tau\left(\bigvee_{j=1}^{m}\bigwedge_{i=1}^{n_j} Q_i^j(p_{Q_i^j}(x))\right) > \theta$$

and where the truth value above is computed according to the min-max semantic, i.e.

$$\tau\left(\bigvee_{j=1}^{m}\bigwedge_{i=1}^{n_j} Q_i^j(p_{Q_i^j}(x))\right) = \max_{j=1,\ldots,m}\left(\min_{i=1,\ldots,n_j}(Q_i^j(p_{Q_i^j}(x))\right). \qquad (4.2)$$

Moreover, it is possible that some of the predicates in $\mathcal{P}$ are either unknown or analytically given and at the same time they can be either unconstrained or constrained to have values higher than a threshold α only within a given subset of $\mathcal{U}$.

Given a set $F=\{\mu_1,\dots,\mu_n\}$ of membership functions associated to the predicates in $\mathcal{P}$, the fuzzy classification system (FCS for short) (4.1) is denoted by S_m^F. The notation $\mathcal{S}_m$ then denotes the collection of all possible FCS's S_m^F, which in turn can be characterized as the collection of all sets F of membership functions. Such FCS's are called *convex fuzzy classification systems*.

As proven in [1], the membership functions in the system can be learned quickly from examples. However, we just use the min-max semantic. In real life, we know that the system semantic in not deterministic. Using our definition, we can extend such a system so to have

$$\tau\left(\bigvee_{j=1}^{m}\bigwedge_{i=1}^{n_j} Q_i^j(p_{Q_i^j}(x))\right) = S_{j=1,\dots,m}(T_{i=1,\dots,n_j}(Q_i^j(p_{Q_i^j}(x)))). \tag{4.3}$$

where S and T are respectively a non deterministic T-conorm and a non deterministic T-norm. The idea behind this extension is clear: during the inference process, the best logical aggregation operator from the classes $\mathcal{S}$ and $\mathcal{T}$ is non-deterministically chosen and applied.

As a consequence, an individual x is *justifiably* classifiable as a positive example for the concept C if

$$\tau\left(\bigvee_{j=1}^{m}\bigwedge_{i=1}^{n_j} Q_i^j(p_{Q_i^j}(x))\right) > \theta$$

for some possible choices of the aggregation operators. Analogously, an individual x is *justifiably* classifiable as a negative example for the concept C if

$$\tau\left(\bigvee_{j=1}^{m}\bigwedge_{i=1}^{n_j} Q_i^j(p_{Q_i^j}(x))\right) \leq \theta$$

for some possible choices of the aggregation operators.

5 Final Comments.

In an attempt to formalize free will and intelligenge power in human decision processes we have introduced the notion of non deterministic aggregation operator. We think that such a notion brings closer theory to practicinners, i.e., to the way practicioners solve in fact problems in real life: freely making decisions and justifying those decisions by means of their expertise (fuzzy membership functions) and global knowledge (aggregation rules).

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ON DEFINING AND COMPUTING FUZZY KERNELS ON $\mathcal{L}$-VALUED SIMPLE GRAPHS

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In this paper we introduce the concept of fuzzy kernels defined on $\mathcal{L}$-valued finite simple graphs in a sense close to fuzzy preference modelling. First we recall the classic concept of kernel associated with a crisp binary relation defined on a finite set. In a second part, we introduce $\mathcal{L}$-fuzzy binary relations. In a third part, we generalize the crisp kernel concept to such $\mathcal{L}$-fuzzy binary relations and in a last part, we present an application to fuzzy choice functions on fuzzy outranking relations.

1 Introduction

In this paper we introduce the concept of fuzzy kernel defined on $\mathcal{L}$-valued binary relations in a sense close to fuzzy preference modelling (cf. [5], [4]). First we introduce the concept of kernel associated to a crisp simple graph. In a second part we introduce in some detail $\mathcal{L}$-valued binary relations where $\mathcal{L}$ is a symmetric evaluation domain for a credibility calculus introduced on binary relation. In a third part we generalize the kernel concept to such $\mathcal{L}$-valued simple graphs, that is $\mathcal{L}$-valued binary relations on finite sets and in a last part, we present the application to fuzzy choice functions on fuzzy outranking relations as used in the context of multicriterian decision aid (cf. [7]).

Let $G(A, R)$ be a simple graph with R being a crisp binary relation on a finite set A of dimension n. A subset Y_R of A is an *dominant (initial) or absorbent (terminal) kernel* of the graph G, if it verifies conjointly the following right and left interior stability and corresponding exterior stability conditions:

right interior stability:

$\forall a, b \in A\ (a \neq b): ((a, b) \in R)) \wedge (b \in Y_R) \Rightarrow (a \notin Y_R).$

left interior stability:

$\forall a, b \in A\ (a \neq b): ((b, a) \in R)) \wedge (b \in Y_R) \Rightarrow (a \notin Y_R).$

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